2012 National Survey of Science and Mathematics Education

Status of High School Mathematics

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Disclaimer

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INTRODUCTION

The 2012 National Survey of Science and Mathematics Education was designed to provide up-to-date information and to identify trends in the areas of teacher background and experience, curriculum and instruction, and the availability and use of instructional resources. A total of 7,752 science and mathematics teachers in schools across the United States participated in this survey, a response rate of 77 percent. The research questions addressed by the study are:

1. To what extent do science and mathematics instruction and ongoing assessment mirror current understanding of learning?

2. What influences teachers' decisions about content and pedagogy?

3. What are the characteristics of the science/mathematics teaching force in terms of race, gender, age, content background, beliefs about teaching and learning, and perceptions of preparedness?

4. What are the most commonly used textbooks/programs, and how are they used?

5. What formal and informal opportunities do science/mathematics teachers have for ongoing development of their knowledge and skills?

6. How are resources for science/mathematics education, including well-prepared teachers and course offerings, distributed among schools in different types of communities and different socioeconomic levels?

The 2012 National Survey is based on a national probability sample of schools and science and mathematics teachers in grades K–12 in the 50 states and the District of Columbia. The sample was designed to allow national estimates of science and mathematics course offerings and enrollment; teacher background preparation; textbook usage; instructional techniques; and availability and use of science and mathematics facilities and equipment. Every eligible school and teacher in the target population had a known, positive probability of being drawn into the sample.

Selecting a random sample of mathematics teachers might result in a smaller than desired number of teachers of advanced mathematics courses. In order to ensure that the sample would include a sufficient number of advanced mathematics teachers for separate analysis, information on teaching assignments was used to create separate domains, and sampling rates were adjusted by domain. This report describes the status of high school (grades 9–12) mathematics instruction based on the responses of 1,822 high school mathematics teachers.[1] For comparison purposes, many of the tables organize these data into three groups based upon the type of a randomly selected class: informal review (i.e., non-college prep mathematics), formal required (e.g.,

[1] A mathematics teacher is defined as someone whose randomly selected class was a grades 9–12 mathematics course.

Algebra I, Algebra II), or formal advanced mathematics (e.g., pre-calculus). Details on which courses comprise each category are included in the Appendix.

Technical detail on the survey sample design, as well as data collection and analysis procedures, is included in the *Report of the 2012 National Survey of Science and Mathematics Education*.[2] The standard errors for the estimates presented in this report are included in parentheses in the tables. The narrative sections of the report generally point out only those differences that are substantial as well as statistically significant at the 0.05 level.[3]

This status report of high school mathematics teaching is organized into major topical areas:

- Characteristics of the high school mathematics teaching force in the United States;
- Professional development of high school mathematics teachers;
- High school mathematics classes offered;
- High school mathematics instruction, in terms of both objectives and class activities;
- Resources available for high school mathematics instruction; and
- Factors affecting high school mathematics instruction.

CHARACTERISTICS OF THE HIGH SCHOOL MATHEMATICS TEACHING FORCE

General Demographics

High school mathematics classes are more likely to be taught by female than male teachers (see Table 1) and the overwhelming majority by white teachers. Table 1 also offers evidence of inequitable assignment of experienced teachers to students. Informal review and formal required classes are more likely than formal advanced classes to be taught by teachers new to teaching mathematics (14 percent compared to 6 percent with two or fewer years of experience). This finding is of interest because of the association between inexperienced teachers and lower student performance.[4]

[2] Banilower, E. R., Smith, P. S., Weiss, I. R., Malzahn, K. A., Campbell, K. M., & Weis, A. M. (2013). *Report of the 2012 national survey of science and mathematics education.* Chapel Hill, NC: Horizon Research, Inc.

[3] The False Discovery Rate was used to control the Type I error rate when comparing multiple groups on the same outcome. Benjamini, Y. & Hochberg, Y. (1995). Controlling the false discovery rate: A practical and powerful approach to multiple testing. *Journal of the Royal Statistical Society*, B, 57, 289–300.

[4] Mumane, R. J., & Phillips, B. R. (1981). What do effective teachers of inner-city children have in common'? *Social Science Research, 10,* 83–100.

Nye, B., Konstantopoulus, S., & Hedges, L.V. (2004). How large are teacher effects? *Educational Evaluation and Policy Analysis, 26* (3), 237–257.

Table 1
Characteristics of the High School Mathematics Teaching Force

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
Sex								
Female	57	(1.6)	60	(4.4)	59	(2.3)	55	(2.1)
Male	43	(1.6)	40	(4.4)	41	(2.3)	45	(2.1)
Race								
White	93	(0.8)	91	(2.5)	91	(1.3)	95	(1.2)
Hispanic or Latino	6	(0.7)	13	(3.3)	5	(1.2)	3	(0.8)
Asian	3	(0.6)	4	(1.8)	2	(0.6)	3	(1.1)
Black or African American	3	(0.5)	4	(1.6)	4	(0.9)	2	(0.5)
American Indian/Alaska Native	0	(0.2)	0	---†	0	(0.4)	0	(0.1)
Native Hawaiian/Other Pacific Islander	0	(0.2)	0	---†	1	(0.3)	0	(0.1)
Two or more races	1	(0.3)	0	(0.4)	2	(0.6)	0	(0.2)
Age								
≤ 30	17	(1.1)	22	(3.8)	21	(1.5)	13	(1.6)
31–40	26	(1.4)	33	(4.7)	23	(1.8)	26	(2.4)
41–50	28	(1.3)	19	(3.4)	28	(2.2)	31	(1.9)
51–60	21	(1.2)	17	(2.9)	21	(1.8)	22	(2.1)
61+	8	(0.8)	9	(2.7)	7	(1.0)	9	(1.4)
Experience Teaching Mathematics at the K–12 Level								
0–2 years	10	(0.8)	14	(2.8)	14	(1.4)	6	(1.2)
3–5 years	13	(1.1)	10	(2.7)	16	(1.8)	11	(1.4)
6–10 years	21	(1.1)	28	(3.6)	19	(1.6)	21	(1.7)
11–20 years	34	(1.2)	34	(3.9)	33	(2.0)	34	(1.9)
≥ 21 years	22	(1.2)	15	(3.0)	18	(1.6)	27	(2.0)
Number of Mathematics Subjects Taught								
1	19	(1.1)	27	(3.4)	21	(1.7)	15	(1.4)
2	48	(1.4)	39	(4.3)	49	(2.5)	50	(1.9)
3 or more	33	(1.4)	34	(4.2)	30	(2.2)	35	(2.0)

† No teachers in the sample selected this response option. Thus, it is not possible to calculate the standard error of this estimate.

About three-quarters of high school mathematics classes are taught by teachers with a college degree in mathematics or mathematics education (see Table 2), and for the vast majority, the teachers have had formal preparation leading to a teaching credential (see Table 3). About half of all high school mathematics classes are taught by teachers who received their teaching credential as part of an undergraduate program. Twenty-one percent of classes are taught by teachers who received certification through a master's program, and an additional 21 percent are taught by teachers who earned a teaching credential through a non-master's post-baccalaureate program. Although differences in teaching experience were evident across course types, preparation patterns appear similar across teachers of varying kinds of secondary mathematics courses.

Table 2
College Degrees Earned by Teachers of High School Mathematics Classes

	Percent of Classes			
	All	Informal Review	Formal Required	Formal Advanced
Mathematics	53 (1.3)	51 (4.9)	53 (2.1)	54 (2.0)
Mathematics Education	57 (1.5)	62 (4.3)	53 (2.2)	59 (2.5)
Mathematics or Mathematics Education	76 (1.4)	77 (4.0)	72 (1.9)	80 (2.1)

Table 3
Paths to Teacher Certification Taken by Teachers of High School Mathematics Classes

	Percent of Classes			
	All	Informal Review	Formal Required	Formal Advanced
An undergraduate program leading to a bachelor's degree and a teaching credential	52 (2.0)	48 (6.4)	54 (3.6)	51 (2.8)
A master's program that also awarded a teaching credential	21 (1.6)	26 (5.7)	17 (2.3)	24 (2.3)
A post-baccalaureate credentialing program (no master's degree awarded)	21 (1.7)	24 (4.7)	23 (2.8)	19 (2.6)
No formal teacher preparation	6 (1.0)	1 (0.9)	7 (1.8)	6 (1.4)

Content Preparedness

As can be seen in Table 4, teachers of high school mathematics classes are similar in terms of preparation in mathematics across course types. Not surprisingly, almost all high school mathematics classes are taught by teachers who have completed some college coursework in mathematics education, and over 80 percent of classes have teachers who were involved in student teaching in mathematics. Ninety-five percent of high school mathematics classes are taught by teachers who have taken a calculus course, 86 percent by teachers with at least one course in statistics, 84 percent by teachers with coursework in linear algebra, and 79 percent by teachers who have taken a course in computer science.

Table 4
Various College Courses Completed by Teachers of High School Mathematics Classes

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
Mathematics Education	90	(1.1)	91	(2.5)	90	(1.5)	89	(1.4)
Student teaching in mathematics	82	(1.2)	82	(3.0)	80	(1.8)	83	(1.5)
Calculus	95	(0.7)	91	(2.5)	94	(1.2)	97	(0.7)
Advanced Calculus	80	(1.4)	72	(4.3)	76	(2.0)	87	(1.8)
Differential Equations	64	(1.8)	57	(4.1)	59	(2.4)	70	(2.5)
Real analysis	45	(1.5)	41	(3.9)	41	(2.0)	51	(2.4)
Linear Algebra	84	(1.2)	80	(3.0)	80	(2.0)	89	(1.4)
Mathematics content for high school teachers	72	(1.6)	77	(3.4)	70	(2.7)	74	(1.7)
Abstract Algebra	71	(1.4)	70	(4.2)	67	(2.1)	75	(1.9)
Axiomatic Geometry (Euclidean or non-Euclidean)	59	(1.5)	64	(4.8)	56	(2.1)	60	(1.8)
Analytic/Coordinate Geometry	55	(1.6)	60	(4.0)	52	(2.1)	57	(2.1)
Integrated Mathematics	35	(1.7)	41	(5.2)	37	(2.6)	32	(2.1)
Statistics	86	(1.1)	87	(3.2)	88	(1.4)	84	(1.8)
Probability	58	(1.5)	58	(4.4)	58	(2.5)	58	(2.2)
Number Theory	55	(1.9)	56	(4.1)	54	(2.7)	56	(2.4)
Discrete Mathematics	54	(1.7)	59	(4.1)	55	(2.5)	53	(2.4)
Other upper division mathematics	45	(1.6)	45	(4.7)	41	(2.3)	49	(2.4)
Computer Science	79	(1.4)	78	(3.8)	78	(2.0)	81	(2.3)
Engineering	19	(1.3)	16	(3.5)	18	(1.7)	20	(2.0)

The survey also asked mathematics teachers to rate how well prepared they feel to teach each of a number of fundamental topics in mathematics. In a large majority of high school mathematics classes, teachers feel very well prepared to teach about algebraic thinking, the number system and operations, and functions (see Table 5). Teachers in over half of high school mathematics classes report being very well prepared to teach modeling, but relatively few classes are taught by teachers who feel very well prepared to teach statistics and probability or discrete mathematics (30 percent and 26 percent, respectively).

Table 5
High School Mathematics Classes in which Teachers Consider Themselves Very Well Prepared to Teach Each of a Number of Topics

	Percent of Classes			
	All	Informal Review	Formal Required	Formal Advanced
Algebraic thinking	92 (0.9)	86 (2.8)	91 (1.4)	95 (0.9)
The number system and operations	91 (0.9)	86 (2.8)	91 (1.2)	93 (1.0)
Functions	86 (1.2)	78 (3.9)	84 (2.0)	91 (1.5)
Measurement	79 (1.2)	68 (4.6)	78 (1.8)	84 (1.4)
Geometry	73 (1.2)	59 (3.8)	77 (2.1)	73 (1.8)
Modeling	59 (1.8)	51 (4.7)	56 (2.3)	64 (2.4)
Statistics and probability	30 (1.2)	26 (3.4)	26 (1.6)	36 (2.0)
Discrete mathematics	26 (1.3)	17 (2.9)	20 (1.7)	33 (2.2)

Data from these items were combined into a composite variable called Perceptions of Preparedness to Teach Mathematics Content.[5] As can be seen in Table 6, teachers of informal review classes feel slightly less well prepared to teach mathematics than teachers of formal mathematics courses.

Table 6
High School Mathematics Class Mean Scores for Teacher Perceptions of Preparedness to Teach Mathematics Content Composite

	Mean Score
All	84 (0.4)
Informal Review	79 (1.5)
Formal Required	83 (0.6)
Formal Advanced	86 (0.6)

Pedagogical Beliefs

Teachers were asked about their beliefs regarding effective teaching and learning in mathematics; results are reported in Table 7. Teachers of high school mathematics classes hold a number of views that align with what is known about effective mathematics instruction. For example, for a large majority of mathematics classes, teachers agree that most class periods should: (1) provide opportunities for students to share their thinking and reasoning, (2) conclude with a summary of the key ideas addressed, and (3) include some review of previously covered ideas and skills.

At the same time, many high school mathematics teachers also hold views that are consistent with traditional mathematics instruction. Teachers in over three-fourths of high school mathematics classes believe that students learn best in classes with students of similar abilities, and that students should be provided with definitions for new vocabulary at the beginning of

[5] The body of this report includes data on selected composite variables. Data for all composite variables are available in the Appendix.

instruction on a mathematical idea. In addition, teachers in only a third of high school mathematics classes agree that teachers should explain an idea to students before having them consider evidence that relates to the idea or agree that hands-on activities/manipulatives should be used primarily to reinforce a mathematical idea that the students have already learned.

Table 7
High School Mathematics Classes in which Teachers Agree† with Various Statements about Teaching and Learning

	Percent of Classes			
	All	Informal Review	Formal Required	Formal Advanced
Most class periods should provide opportunities for students to share their thinking and reasoning	93 (0.9)	94 (2.2)	94 (1.1)	92 (1.4)
Most class periods should conclude with a summary of the key ideas addressed	90 (1.0)	92 (2.1)	92 (1.4)	88 (1.6)
Most class periods should include some review of previously covered ideas and skills	86 (1.0)	91 (2.3)	86 (1.5)	84 (1.5)
Students should be provided with the purpose for a lesson as it begins	85 (1.0)	92 (2.2)	86 (1.8)	81 (1.5)
Students should be assigned homework most days	83 (1.2)	76 (4.0)	85 (1.5)	82 (1.8)
At the beginning of instruction on a mathematical idea, students should be provided with definitions for new vocabulary that will be used	81 (1.0)	82 (3.0)	83 (1.4)	79 (1.7)
It is better for mathematics instruction to focus on ideas in depth, even if that means covering fewer topics	79 (1.2)	82 (3.9)	77 (1.7)	79 (1.8)
Students learn mathematics best in classes with students of similar abilities	76 (1.1)	73 (3.8)	73 (1.7)	80 (1.7)
Inadequacies in students' mathematics background can be overcome by effective teaching	76 (1.4)	72 (3.3)	77 (1.9)	76 (2.1)
Teachers should explain an idea to students before having them investigate the idea	36 (1.6)	39 (3.8)	38 (2.5)	33 (1.9)
Hands-on activities/manipulatives should be used primarily to reinforce a mathematical idea that the students have already learned	36 (1.5)	43 (4.5)	38 (2.2)	31 (2.0)

† Includes teachers indicating "strongly agree" or "agree" on a 5-point scale ranging from 1 "strongly disagree" to 5 "strongly agree."

Pedagogical Preparedness

The survey asked teachers two series of items focused on their preparedness for a number of tasks associated with instruction. First, they were asked how well prepared they feel to address diverse learners in their instruction. Second, they were asked how well prepared they feel to monitor and address student understanding, focusing on a specific unit in the randomly selected class.

As can be seen in Table 8, high school mathematics teachers in the majority of classes feel very well prepared to manage classroom discipline and encourage the participation of females in mathematics. For a somewhat smaller percentage of classes, mathematics teachers feel very well prepared to encourage students' interest in mathematics, to encourage participation of students from low socioeconomic background in mathematics, and to encourage participation of racial or

ethnic minorities in mathematics. Teachers in only about one-third of high school mathematics classes feel very well prepared to differentiate instruction. Teachers of informal review courses are more likely than teachers of formal courses to feel very well prepared to teach mathematics to students with learning disabilities.

Table 8
High School Mathematics Classes in which Teachers Consider Themselves Very Well Prepared for Each of a Number of Tasks

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
Manage classroom discipline	59	(2.0)	60	(5.5)	59	(3.1)	58	(3.2)
Encourage participation of females in mathematics	52	(2.4)	55	(5.5)	49	(3.2)	55	(3.4)
Encourage students' interest in mathematics	40	(2.3)	36	(5.6)	37	(3.1)	44	(3.3)
Encourage participation of students from low socioeconomic backgrounds in mathematics	40	(2.2)	44	(5.7)	37	(3.0)	41	(3.3)
Encourage participation of racial or ethnic minorities in mathematics	40	(2.0)	43	(5.6)	39	(3.1)	39	(3.1)
Plan instruction so students at different levels of achievement can increase their understanding of the ideas targeted in each activity	32	(2.1)	34	(6.0)	33	(2.6)	30	(2.6)
Provide enrichment opportunities for gifted students	24	(2.1)	24	(5.3)	19	(2.6)	31	(2.8)
Teach mathematics to students who have learning disabilities	19	(1.7)	33	(6.0)	16	(2.5)	16	(2.2)
Teach mathematics to students who have physical disabilities	16	(1.4)	21	(4.3)	14	(2.2)	16	(2.2)
Teach mathematics to English-language learners	13	(1.4)	16	(4.2)	13	(2.2)	13	(2.0)

Table 9 shows the percentage of classes taught by teachers who feel very well prepared for each of a number of tasks related to instruction. In the majority of high school mathematics classes, teachers feel very well prepared to assess student understanding at the end of a unit and monitor student understanding during instruction. Teachers feel very well prepared to find out what students thought or already knew about the key mathematical ideas in less than half (48 percent) of high school mathematics classes. Teachers of formal advanced mathematics courses are more likely than teachers of informal review courses to feel very well prepared to implement the mathematics textbook/program.

Table 9
High School Mathematics Classes in which Teachers Feel Very Well Prepared for Each of a Number of Tasks in the Most Recent Unit

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
Assess student understanding at the conclusion of this unit	71	(1.5)	66	(4.5)	72	(2.0)	71	(2.4)
Monitor student understanding during this unit	64	(1.7)	62	(4.6)	64	(2.2)	65	(2.3)
Implement the mathematics textbook/program to be used during this unit	61	(1.9)	48	(5.8)	59	(3.1)	66	(2.6)
Anticipate difficulties that students will have with particular mathematical ideas and procedures in this unit	59	(1.4)	58	(4.6)	58	(2.1)	61	(2.3)
Find out what students thought or already knew about the key mathematical ideas	48	(1.6)	53	(4.5)	45	(2.4)	49	(2.3)

† This item was presented only to teachers who indicated using commercially published textbooks/programs in the most recent unit.

PROFESSIONAL DEVELOPMENT OF HIGH SCHOOL MATHEMATICS TEACHERS

One important measure of teachers' continuing education is how long it has been since they participated in professional development. Teachers of almost 90 percent of high school mathematics classes have participated in mathematics-focused professional development (i.e., focused on mathematics content or the teaching of mathematics) within the last three years (see Table 10).

Table 10
Most Recent Mathematics-Focused† Professional Development Completed by Teachers of High School Mathematics Classes

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
In the last 3 years	89	(1.0)	92	(2.4)	90	(1.5)	88	(1.6)
4–6 years ago	6	(0.7)	5	(2.1)	5	(0.9)	7	(1.3)
7–10 years ago	1	(0.4)	1	(0.4)	2	(0.6)	1	(0.4)
More than 10 years ago	1	(0.3)	0	(0.4)	1	(0.3)	2	(0.5)
Never	2	(0.5)	2	(1.0)	3	(0.9)	2	(0.7)

† Includes professional development focused on mathematics or mathematics teaching.

However, teachers of high school mathematics classes report limited extent of participation in professional development specific to mathematics teaching. Teachers in only a third of high school mathematics classes have completed more than 35 hours in mathematics-related professional development in the last three years (see Table 11).

Table 11
Time Spent on Professional Development
in the Last Three Years by Teachers of High School Mathematics Classes

	Percent of Classes			
	All	Informal Review	Formal Required	Formal Advanced
Less than 6 hours	20 (1.2)	25 (3.6)	18 (1.8)	21 (1.7)
6–15 hours	24 (1.5)	23 (3.8)	26 (2.1)	23 (2.2)
16–35 hours	22 (1.0)	22 (3.5)	21 (1.7)	23 (1.7)
More than 35 hours	33 (1.6)	30 (3.8)	34 (2.4)	32 (2.1)

As to how this time is spent, the workshop is the most common form of professional development, with teachers in 90 percent of high school mathematics classes having attended at least one workshop in the previous three years (see Table 12). Teachers in three-fourths of high school mathematics classes have participated in a professional learning community or other type of teacher study group within the past three years.

Table 12
High School Mathematics Classes in which Teachers Participated
in Various Professional Development Activities in the Last Three Years

	Percent of Classes			
	All	Informal Review	Formal Required	Formal Advanced
Attended a workshop on mathematics or mathematics teaching	90 (1.0)	86 (3.4)	91 (1.5)	90 (1.4)
Participated in a professional learning community/lesson study/teacher study group focused on mathematics or mathematics teaching	75 (1.9)	77 (3.3)	77 (2.5)	71 (2.5)
Received feedback about your mathematics teaching from a mentor/coach formally assigned by the school/district/diocese[†]	54 (2.2)	58 (6.0)	58 (3.6)	49 (2.9)
Attended a national, state, or regional mathematics teacher association meeting	39 (1.6)	43 (4.5)	39 (2.4)	37 (2.2)

[†] This item was asked of all teachers whether or not they had participated in professional development in the last three years.

The emerging consensus about effective professional development suggests that teachers need opportunities to work with colleagues who face similar challenges, such as other teachers from their school or those who have similar teaching assignments. Other recommendations include engaging teachers in investigations, both to learn disciplinary content and to experience inquiry-oriented learning; to examine student work and other classroom artifacts for evidence of what students do and do not understand; and to apply what they have learned in their classrooms and subsequently discuss how it went.[6] Accordingly, teachers who had participated in professional

[6] Elmore, R. F. (2002). *Bridging the gap between standards and achievement: The imperative for professional development in education.* Washington, DC: Albert Shanker Institute.

Garet, M. S., Porter, A. C., Desimone, L., Birman, B. F., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal, 38*(4), 915–945.

development in the last three years were asked a series of additional questions about the nature of those experiences.

As can be seen in Table 13, in well over half of high school mathematics classrooms, teachers have had substantial opportunity to work closely with other teachers from their school and with teachers at other schools teaching the same grade and subject. Relatively few mathematics classes are taught by teachers who have had opportunities to examine classroom artifacts as part of their professional development. Teachers of informal review classes, compared to teachers of formal advanced courses, have had more opportunities to try out what they learned in the classroom and then talk about it as a part of the professional development.

Table 13
High School Mathematics Classes in which Teachers' Professional Development in the Last Three Years Had Each of a Number of Characteristics to a Substantial Extent[†]

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
You worked closely with other mathematics teachers from your school	67	(2.4)	76	(5.4)	69	(3.7)	64	(2.7)
You worked closely with other mathematics teachers who taught the same grade and/or subject whether or not they were from your school	58	(2.4)	66	(6.9)	58	(3.8)	55	(3.3)
You had opportunities to try out what you learned in your classroom and then talk about it as part of the professional development	47	(2.6)	60	(6.7)	52	(4.0)	40	(2.9)
You had opportunities to engage in mathematics investigations	42	(2.1)	44	(7.1)	47	(3.3)	38	(2.8)
You had opportunities to examine classroom artifacts (e.g., student work samples)	37	(2.5)	45	(7.4)	40	(3.7)	31	(2.9)
The professional development was a waste of your time	7	(1.0)	5	(2.0)	6	(1.6)	8	(1.2)

[†] Includes teachers indicating 4 or 5 on a 5-point scale ranging from 1 "Not at all" to 5 "To a great extent."

College courses have the potential to address content in more depth than may be possible in other professional development venues, such as workshops. As another indicator of the extent to which teachers are staying current in their field, the 2012 National Survey asked teachers when they had last taken a formal course for college credit in mathematics and how to teach mathematics. As can be seen in Table 14, teachers in nearly 40 percent of high school mathematics classes have not taken a course for college credit in either mathematics or the teaching of mathematics in the last 10 years. However, teachers of informal review classes and teachers of formal required courses are more likely than teachers of formal advanced courses to have taken such a course in the last three years.

Table 14
Most Recent College Coursework by Teachers of High School Mathematics Classes

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
Mathematics								
In the last 3 years	19	(1.2)	21	(3.6)	24	(1.9)	14	(1.3)
4–6 years ago	19	(1.2)	23	(3.4)	20	(1.9)	16	(1.5)
7–10 years ago	16	(1.1)	19	(4.0)	14	(1.6)	17	(1.7)
More than 10 years ago	46	(1.8)	36	(3.8)	41	(2.5)	53	(2.3)
Never	0	(0.1)	1	(0.6)	0	(0.3)	0	(0.1)
The Teaching of Mathematics								
In the last 3 years	21	(1.1)	28	(3.5)	25	(1.8)	15	(1.5)
4–6 years ago	14	(1.0)	13	(3.2)	14	(1.6)	14	(1.7)
7–10 years ago	14	(1.0)	13	(3.4)	13	(1.4)	15	(1.3)
More than 10 years ago	41	(1.4)	36	(3.6)	39	(2.1)	44	(2.0)
Never	10	(1.1)	9	(2.6)	10	(1.5)	11	(1.4)
Mathematics or the Teaching of Mathematics								
In the last 3 years	27	(1.3)	34	(4.1)	31	(1.9)	22	(1.7)
4–6 years ago	19	(1.1)	16	(3.2)	20	(1.8)	19	(1.9)
7–10 years ago	15	(1.0)	15	(4.0)	13	(1.5)	16	(1.3)
More than 10 years ago	39	(1.4)	34	(3.9)	36	(2.3)	43	(2.0)
Never	0	(0.1)	1	(0.6)	0	(0.3)	0	(0.1)

Another series of items asked about the focus of the opportunities teachers had to learn about content and the teaching of that content in the last three years, whether through professional development or college coursework. Teachers of about half of high school mathematics classes have had professional growth opportunities that gave heavy emphasis to learning how to use hands-on activities or manipulatives for mathematics instruction, planning instruction for students at different levels of achievement, assessing students at the end of instruction, and monitoring student understanding during instruction (see Table 15). Teachers in relatively few high school mathematics classes indicated a heavy emphasis on providing enrichment experiences for gifted students or teaching mathematics to English-language learners.

Table 15
High School Mathematics Classes in which Teachers' Professional Development/ Coursework in the Last Three Years Gave Heavy Emphasis† to Various Areas

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
Learning how to use hands-on activities/manipulatives for mathematics instruction	55	(2.1)	59	(6.2)	52	(2.9)	56	(3.0)
Planning instruction so students at different levels of achievement can increase their understanding of the ideas targeted in each activity	51	(2.4)	47	(6.1)	51	(3.3)	52	(3.5)
Assessing student understanding at the conclusion of instruction on a topic	49	(2.3)	47	(6.3)	52	(3.7)	46	(2.9)
Monitoring student understanding during mathematics instruction	49	(2.2)	57	(6.1)	49	(3.0)	45	(3.1)
Learning about difficulties that students may have with particular mathematical ideas and procedures	44	(2.1)	45	(6.3)	45	(3.3)	41	(2.8)
Deepening your own mathematics content knowledge	34	(2.1)	41	(6.4)	33	(3.0)	33	(2.5)
Finding out what students think or already know about the key mathematical ideas prior to instruction on those ideas	31	(1.9)	39	(5.8)	31	(2.7)	29	(2.7)
Providing alternative mathematics learning experiences for students with special needs	30	(1.8)	37	(5.9)	31	(2.6)	27	(3.0)
Implementing the mathematics textbook/program to be used in your classroom	30	(1.8)	27	(5.1)	36	(3.0)	23	(2.5)
Providing enrichment experiences for gifted students	22	(1.8)	19	(4.3)	22	(2.6)	23	(2.9)
Teaching mathematics to English-language learners	17	(1.6)	21	(5.0)	20	(2.4)	13	(2.1)

† Includes teachers responding 4 or 5 on a 5-point scale ranging from 1 "Not at all" to 5 "To a great extent."

In addition to asking teachers about their involvement as participants in professional development, the survey asked teachers whether they had served in various leadership roles in the profession in the last three years. As can be seen in Table 16, teachers in only about 1 in 4 high school mathematics classes supervised a student teacher or led a teacher study group. Few teachers of high school mathematics classes have led an in-service workshop.

Table 16
High School Mathematics Classes in which Teachers Served in Various Leadership Roles in the Last Three Years

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
Supervised a student teacher	26	(2.0)	31	(5.2)	23	(3.2)	27	(3.2)
Led a teacher study group focused on mathematics teaching	25	(1.8)	25	(4.7)	25	(3.1)	25	(2.3)
Served as a formally assigned mentor/coach for mathematics teaching	22	(1.7)	32	(5.0)	20	(2.7)	21	(2.4)
Taught in-service workshops on mathematics or mathematics teaching	14	(1.4)	18	(4.6)	11	(2.0)	15	(1.8)

HIGH SCHOOL MATHEMATICS CLASSES OFFERED

Of the high schools (schools including grades 9, 10, 11, and 12) in the United States, nearly all offer at least one formal/college-prep mathematics course such as Algebra 1 (see Table 17). An overwhelming majority offer higher level formal/college courses (Levels 2, 3, and 4) as well. Nearly 80 percent of high schools offer a non-college prep course, such as Remedial Math.

About three-quarters of high schools offer courses that might qualify for college credit, such as Advanced Placement (AP) classes, with 52 percent offering AP Calculus AB, 27 percent offering AP Statistics, and 23 percent offering AP Calculus BC. There is a fairly large disparity between the percentage of high schools offering each AP mathematics course and the percentage of high school students with access to the course, most likely due to the fact that large schools are more likely than small ones to offer advanced mathematics courses, and that small schools outnumber large schools in the United States.

Table 17
Availability of Mathematics Courses at High Schools

	Percent of High Schools Offering	Percent of High School Students with Access
Non-college prep (e.g., Remedial Math, General Math, Consumer Math)	78 (3.2)	83 (1.8)
Formal/College-prep Level 1 (e.g., Algebra 1, Integrated Math 1)	99 (0.7)	100 (0.2)
Formal/College-prep Level 2 (e.g., Geometry, Integrated Math 2)	90 (3.7)	99 (0.3)
Formal/College-prep Level 3 (e.g., Algebra 2, Algebra and Trigonometry)	94 (3.5)	99 (0.3)
Formal/College-prep Level 4 (e.g., Pre-Calculus, Algebra 3)	85 (3.8)	98 (0.4)
Courses that might qualify for college credit (e.g., AP Calculus, AP Statistics)	76 (4.0)	94 (0.8)
AP Calculus AB	52 (3.5)	81 (1.6)
AP Statistics	27 (2.1)	59 (1.9)
AP Calculus BC	23 (2.5)	47 (2.1)

The typical high school mathematics class has approximately 21 students; two-thirds of the classes have between 14 and 28 students. On average, class sizes across course types differ by only a few students; informal review, formal required, and formal advanced classes have mean class sizes of 19, 22, and 21 students, respectively. Across all classes, 48 percent of mathematics students are female; a greater proportion of students in formal mathematics classes than informal review classes are female (see Table 18). Relatively few students from race/ethnic groups historically underrepresented[7] in mathematics take advanced mathematics courses. Ability grouping is a common practice in mathematics education. Not surprisingly, formal advanced classes are far more likely to be composed of high-achieving students, and informal review

[7] Includes students identifying themselves as American Indian or Alaskan Native, Black, Hispanic or Latino, or Native Hawaiian or Other Pacific Islander.

classes are more likely to be composed of mostly low-achieving students (see Table 19). Heterogeneous grouping of students is most common in the formal required courses.

Table 18
Demographics of Students in High School Mathematics Courses

	Percent of Students	
	Female	Historically Underrepresented
All	48 (0.7)	31 (1.1)
Informal Review	42 (1.4)	45 (3.3)
Formal Required	49 (1.0)	35 (1.6)
Formal Advanced	49 (1.1)	23 (1.3)

Table 19
Prior-Achievement Grouping in High School Mathematics Classes

	Percent of Classes			
	All	Informal Review	Formal Required	Formal Advanced
Mostly low achievers	24 (1.2)	72 (3.7)	26 (2.1)	8 (1.4)
Mostly average achievers	28 (1.7)	14 (3.4)	35 (2.4)	27 (2.3)
Mostly high achievers	26 (1.2)	2 (1.3)	14 (1.4)	45 (2.3)
A mixture of levels	21 (1.2)	12 (2.4)	25 (1.7)	20 (1.9)

HIGH SCHOOL MATHEMATICS INSTRUCTION

The next three sections draw on teachers' descriptions of what transpires in mathematics classrooms, in terms of teachers' autonomy for making decisions regarding the content and pedagogy of their classes, instructional objectives, and class activities.

Teachers' Perceptions of their Decision Making Autonomy

Teachers were asked the extent to which they had control over a number of curriculum and instruction decisions for their classes. In high school mathematics classes, teachers are more likely to perceive themselves as having strong control over pedagogical decisions such as determining the amount of homework to be assigned, selecting teaching techniques, and choosing criteria for grading student performance (see Table 20). In fewer classes, teachers perceive themselves as having strong control in determining course goals and objectives, selecting what content/skills to teach, and selecting textbooks/programs.

Table 20
High School Mathematics Classes in which Teachers Report Having Strong Control Over Various Curriculum and Instruction Decisions

	Percent of Classes			
	All	Informal Review	Formal Required	Formal Advanced
Determining the amount of homework to be assigned	75 (2.1)	68 (5.9)	70 (3.0)	82 (2.3)
Selecting teaching techniques	72 (1.9)	65 (5.2)	65 (3.1)	81 (2.0)
Choosing criteria for grading student performance	55 (2.2)	53 (6.6)	48 (3.5)	63 (2.9)
Determining course goals and objectives	28 (2.2)	33 (6.4)	19 (2.8)	34 (2.9)
Selecting content, topics, and skills to be taught	24 (1.9)	32 (6.6)	14 (2.3)	31 (2.6)
Selecting textbooks/programs	20 (2.1)	23 (5.0)	12 (2.6)	26 (3.3)

These items were combined into two composite variables: Curriculum Control and Pedagogical Control. It appears high school mathematics teachers, overall, perceive greater control over pedagogical decisions compared to curriculum decisions. Comparing different types of classes, teachers of formal required classes perceive a lower level of curriculum control than teachers of either informal review or formal advanced classes (see Table 21). One possible explanation is that formal required courses, which are tied most closely to high-stakes accountability testing, call for teachers to follow a more prescribed curriculum than other course types.

Table 21
High School Mathematics Class Mean Scores for Curriculum Control and Pedagogical Control Composites

	Mean Score	
	Pedagogical Control	Curriculum Control
All	88 (0.7)	52 (1.4)
Informal Review	86 (2.2)	56 (4.3)
Formal Required	85 (1.1)	44 (2.1)
Formal Advanced	91 (0.8)	57 (2.0)

Instructional Objectives

Teachers were given a list of potential objectives and asked to rate each in terms of the emphasis they receive in the randomly selected class. As can be seen in Table 22, a majority of high school mathematics classes have a heavy emphasis on learning mathematical practices, understanding mathematical ideas, and preparing for further study in mathematics. The latter two are more strongly emphasized in formal required or advanced classes than in informal review classes. Informal review classes have a greater emphasis than formal advanced classes on learning test tasking skills/strategies. These differences suggest that informal review classes devote more attention to preparing students for standardized testing for its own sake, whereas formal required and advanced courses focus more attention on deepening students' understanding of mathematics. Less than a third of high school mathematics classes give heavy emphasis to either learning to perform computations with speed and accuracy or increasing students' interest in mathematics.

Table 22
High School Mathematics Classes with Heavy Emphasis on Various Instructional Objectives

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
Understanding mathematical ideas	69	(1.4)	54	(4.2)	70	(2.2)	73	(1.9)
Preparing for further study in mathematics	54	(1.6)	35	(4.1)	52	(2.3)	62	(2.4)
Learning mathematical practices (e.g., considering how to approach a problem, justifying solutions)	54	(1.4)	49	(4.1)	54	(2.3)	56	(2.0)
Learning mathematical procedures and/or algorithms	48	(1.6)	46	(4.6)	45	(2.6)	52	(2.4)
Learning about real-life applications of mathematics	28	(1.3)	36	(4.7)	26	(1.8)	28	(1.9)
Learning test taking skills/strategies	28	(1.3)	37	(4.9)	30	(2.2)	23	(1.6)
Increasing students' interest in mathematics	27	(1.5)	29	(3.9)	24	(2.0)	29	(2.2)
Learning to perform computations with speed and accuracy	17	(1.1)	16	(3.1)	15	(1.7)	20	(1.7)

Class Activities

The 2012 National Survey included several items that provide information about how mathematics is taught at the high school level. One series of items listed various instructional strategies and asked teachers to indicate the frequency with which they used each in a randomly selected class. As can be seen in Table 23, the vast majority of high school mathematics classes include on a weekly basis the teacher explaining mathematical ideas, whole class discussions, and students explaining and justifying their method for solving a problem. About 6 in 10 classes have students working in small groups and considering multiple representations in solving a problem at least once a week. A similar proportion requires students to compare and contrast different methods for solving a problem. Taking short-answer tests and quizzes and practicing for standardized tests on a weekly basis are more commonly found in informal review classes than in formal advanced classes, further supporting the notion that informal review classes emphasize test preparation more than other types of classes.

Table 23
High School Mathematics Classes in which Teachers Report Using Various Activities at Least Once a Week

	Percent of Classes			
	All	Informal Review	Formal Required	Formal Advanced
Explain mathematical ideas to the whole class	95 (0.7)	90 (2.9)	95 (1.2)	97 (0.8)
Engage the whole class in discussions	84 (1.1)	83 (3.5)	85 (2.0)	83 (1.8)
Have students explain and justify their method for solving a problem	79 (1.3)	75 (3.8)	81 (2.3)	78 (2.0)
Have students consider multiple representations in solving a problem (e.g., numbers, tables, graphs, pictures)	64 (1.5)	63 (4.4)	65 (2.4)	64 (2.3)
Have students work in small groups	62 (1.7)	66 (3.7)	63 (2.3)	61 (2.3)
Give tests and/or quizzes that include constructed-response/open-ended items	56 (1.6)	47 (4.0)	57 (2.2)	59 (2.2)
Have students compare and contrast different methods for solving a problem	55 (1.6)	56 (4.5)	58 (2.3)	53 (2.0)
Have students present their solution strategies to the rest of the class	46 (1.5)	47 (3.5)	49 (2.3)	43 (1.9)
Give tests and/or quizzes that are predominantly short-answer (e.g., multiple choice, true/false, fill in the blank)	36 (1.2)	44 (4.4)	41 (2.0)	29 (1.9)
Have students practice for standardized tests	31 (1.5)	42 (4.6)	33 (2.1)	26 (2.2)
Have students read from a mathematics textbook/program or other mathematics-related material in class, either aloud or to themselves	25 (1.5)	33 (4.2)	26 (2.0)	22 (1.9)
Provide manipulatives for students to use in problem-solving/investigations	18 (1.1)	29 (3.5)	21 (1.8)	11 (1.1)
Have students develop mathematical proofs	17 (1.1)	8 (2.1)	24 (1.8)	12 (1.1)
Focus on literacy skills (e.g., informational reading or writing strategies)	14 (1.0)	19 (3.2)	14 (1.3)	12 (1.2)
Have students write their reflections (e.g., in their journals) in class or for homework	10 (1.0)	15 (3.4)	12 (1.5)	7 (0.9)
Have students attend presentations by guest speakers focused on mathematics in the workplace	1 (0.3)	4 (2.0)	1 (0.2)	1 (0.4)

Overall, high school mathematics classes utilize calculators more frequently than other instructional technologies, such as the Internet, computers, or classroom response systems. As can be seen in Table 24, 85 percent of formal advanced mathematics classes use graphing calculators at least weekly (compared to 37 and 48 percent of informal review or formal required classes, respectively). Informal review classes use personal computers, including laptops, more frequently than formal required or formal advanced classes.

Table 24
High School Mathematics Classes in which Teachers Report that Students Use Various Instructional Technologies at Least Once a Week

	Percent of Classes			
	All	Informal Review	Formal Required	Formal Advanced
Graphing calculators	64 (2.1)	37 (5.0)	48 (3.4)	85 (1.8)
Scientific calculators	52 (2.2)	61 (6.8)	59 (3.2)	43 (3.7)
Four-function calculators	33 (2.3)	44 (6.6)	38 (3.5)	25 (3.3)
Internet	11 (1.3)	23 (5.6)	9 (1.7)	11 (1.7)
Personal computers, including laptops	10 (1.3)	24 (5.8)	9 (1.9)	8 (1.4)
Classroom response system or "Clickers"	4 (0.7)	5 (2.3)	6 (1.4)	3 (0.7)
Hand-held computers	4 (0.9)	6 (3.4)	4 (1.3)	3 (0.9)
Probes for collecting data	1 (0.4)	2 (1.7)	1 (0.7)	1 (0.3)

In addition to asking about class activities in the course as a whole, the 2012 National Survey asked teachers about activities that took place during their most recent mathematics lesson in the randomly selected class. Over 90 percent of high school mathematics lessons include the teacher explaining a mathematical idea to the whole class (see Table 25). Whole class discussion and students completing textbook/worksheet problems are also very common in mathematics lessons. Hands-on/manipulative activities are less common in formal advanced mathematics classes compared to others (15 percent vs. 27 or 25 percent of the most recent lessons).

Table 25
High School Mathematics Classes Participating in Various Activities in the Most Recent Lesson

	Percent of Classes			
	All	Informal Review	Formal Required	Formal Advanced
Teacher explaining a mathematical idea to the whole class	94 (0.8)	94 (2.2)	94 (1.2)	95 (0.9)
Students completing textbook/worksheet problems	83 (1.1)	85 (3.1)	84 (1.6)	80 (1.6)
Whole class discussion	75 (1.3)	72 (4.1)	76 (2.2)	74 (2.0)
Teacher conducting a demonstration while students watched	65 (1.2)	72 (3.8)	66 (2.1)	62 (2.2)
Students using instructional technology	43 (1.3)	39 (4.4)	39 (2.1)	49 (2.1)
Students doing hands-on/manipulative activities	21 (1.3)	27 (3.2)	25 (2.1)	15 (1.8)
Test or quiz	20 (1.4)	19 (3.3)	21 (1.8)	20 (1.8)
Students reading about mathematics	16 (1.2)	17 (3.3)	17 (2.0)	15 (1.7)
Practicing for standardized tests	16 (1.1)	22 (3.7)	18 (1.7)	13 (1.4)

The survey also asked teachers to estimate the time spent on each of a number of types of activities in the most recent mathematics lesson. Overall, 48 percent of class time is spent on whole class activities, 22 percent on small group work, and 22 percent on students working individually (see Table 26). Non-instructional activities, including attendance taking and interruptions, account for 9 percent of mathematics class time. On average, whole group activities are somewhat more prevalent in formal advanced mathematics classes, and students working individually is somewhat more prevalent in informal review mathematics classes.

Table 26
Average Percentage of Time Spent on Different Activities in the Most Recent High School Mathematics Lesson

	Average Percent of Class Time							
	All		Informal Review		Formal Required		Formal Advanced	
Whole class activities (e.g., lectures, explanations, discussions)	48	(0.7)	40	(1.8)	47	(1.1)	51	(1.0)
Small group work	22	(0.9)	22	(1.7)	22	(1.1)	21	(1.3)
Students working individually (e.g., reading textbooks, completing worksheets, taking a test or quiz)	22	(0.6)	29	(1.8)	22	(0.9)	19	(1.0)
Non-instructional activities (e.g., attendance taking, interruptions)	9	(0.2)	9	(0.5)	9	(0.3)	8	(0.2)

Homework and Assessment Practices

Teachers were asked about the amount of homework assigned per week in the randomly selected class. As can be seen in Table 27, half of high school mathematics classes assign between 31 and 90 minutes of homework per week. Overall, there is a trend of more homework in advanced mathematics classes than other mathematics classes.

Table 27
Amount of Homework Assigned in High School Mathematics Classes per Week

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
Fewer than 15 minutes per week	7	(1.1)	20	(4.9)	5	(1.4)	4	(1.2)
15–30 minutes per week	8	(1.3)	13	(3.9)	9	(1.9)	5	(1.3)
31–60 minutes per week	23	(1.8)	29	(5.4)	26	(2.9)	18	(2.3)
61–90 minutes per week	27	(1.8)	21	(5.1)	28	(2.5)	28	(3.0)
91–120 minutes per week	12	(1.2)	7	(3.0)	13	(2.0)	14	(1.6)
More than 120 minutes per week	23	(1.9)	9	(3.9)	19	(2.5)	32	(3.5)

Teachers were also given a list of ways that they might assess student progress and asked to describe which practices they used in the most recently completed unit in the randomly selected class. The vast majority of high school mathematics classes included informal assessment practices during the unit to see if students were "getting it" (see Table 28). For example, 97 percent of high school mathematics classes involved the teacher questioning students during activities to monitor understanding. Using whole class informal assessments such as "thumbs up/thumbs down" was another common practice, used in 83 percent of mathematics classes.

In addition, the use of formal assessment techniques such as grading student work, quizzes, and tests, as well as reviewing correct answers to assignments were also prevalent features of mathematics units. Teachers in more than 9 out of 10 high school mathematics classes administered a test or quiz to assign grades and reviewed student work in the most recent unit.

Assessing or probing student thinking at the beginning of a unit was included in about 4 in 10 high school mathematics classes; this practice was more common in informal review classes compared to formal required classes, and more common in formal required classes than in formal advanced classes.

Table 28
High School Mathematics Classes in which Teachers Report Assessing Students Using Various Methods in the Most Recent Unit

	Percent of Classes			
	All	Informal Review	Formal Required	Formal Advanced
Questioned individual students during class activities to see if they were "getting it"	97 (0.5)	94 (2.4)	97 (0.8)	97 (0.7)
Reviewed student work (e.g., homework, notebooks, journals, portfolios, projects) to see if they were "getting it"	96 (0.7)	97 (1.6)	98 (0.7)	93 (1.1)
Administered one or more quizzes and/or tests to assign grades	94 (0.6)	86 (3.4)	96 (0.8)	96 (0.8)
Went over the correct answers to assignments, quizzes, and/or tests with the class as a whole	92 (0.7)	87 (2.5)	93 (1.0)	93 (1.0)
Administered one or more quizzes and/or tests to see if students were "getting it"	85 (1.4)	85 (3.2)	87 (1.7)	83 (2.2)
Assigned grades to student work (e.g., homework, notebooks, journals, portfolios, projects)	85 (1.0)	82 (3.6)	89 (1.4)	81 (1.7)
Used information from informal assessments of the entire class (e.g., asking for a show of hands, thumbs up/thumbs down, clickers, exit tickets) to see if students were "getting it"	83 (1.1)	81 (3.1)	86 (1.7)	80 (1.6)
Administered an assessment, task, or probe at the beginning of the unit to find out what students thought or already knew about the key mathematical ideas	42 (1.8)	59 (3.9)	44 (2.5)	34 (2.6)
Had students use rubrics to examine their own or their classmates' work	8 (0.8)	11 (2.7)	8 (1.3)	6 (0.7)

The survey asked how often students in the randomly selected class were required to take assessments the teachers did not develop, such as state or district benchmark assessments. Nearly 80 percent of high school mathematics classes are required to take such an assessment at least once a year (see Table 29). Additionally, administration of three or more external assessments is more common in informal review and formal required classes than formal advanced classes.

Table 29
Frequency of Required External Testing in High School Mathematics Classes

	Percent of Classes			
	All	Informal Review	Formal Required	Formal Advanced
Never	22 (1.3)	21 (3.2)	9 (1.4)	34 (1.9)
Once a year	28 (1.3)	26 (3.8)	28 (2.1)	29 (1.9)
Twice a year	15 (1.1)	13 (3.0)	16 (1.5)	15 (1.7)
Three or four times a year	22 (1.3)	25 (3.7)	27 (2.2)	15 (1.3)
Five or more times a year	13 (1.0)	16 (4.0)	20 (1.7)	7 (0.9)

RESOURCES AVAILABLE FOR HIGH SCHOOL MATHEMATICS INSTRUCTION

Instructional Materials

The 2012 National Survey collected data on the use of commercially published instructional materials in mathematics classes. Approximately 80 percent of high school mathematics classes use commercially published materials (see Table 30). Commercially published materials are more common in formal advanced mathematics classes than in formal required or informal review classes.

Table 30
High School Mathematics Classes Using Commercially Published Instructional Materials

	Percent of Classes
All	81 (1.1)
Informal Review	74 (3.7)
Formal Required	78 (1.8)
Formal Advanced	86 (1.4)

The survey also asked if one textbook/program is used all or most of the time, or if multiple materials are used. About two-thirds of all high school mathematics classes use a single textbook or program (see Table 31). Interestingly, formal advanced classes are more likely to use a single textbook/program than formal required classes; informal review classes are the least likely to use a single textbook/program.

Table 31
Instructional Materials Used in High School Mathematics Classes

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
One commercially published textbook or program most of the time	65	(1.5)	46	(4.3)	61	(2.4)	75	(1.8)
Multiple commercially published textbooks/programs most of the time	16	(1.0)	28	(3.7)	18	(1.7)	11	(1.3)
Non-commercially published instructional materials most of the time	19	(1.1)	26	(3.7)	22	(1.8)	14	(1.4)

Teachers who indicated that the randomly selected class used commercially published materials were asked to record the title, author, year, and ISBN of the material used most often in the class. Using this information, the publisher of the materials was identified. Table 32 shows the most commonly used high school mathematics materials.

Table 32
Most Commonly Used High School Mathematics Textbooks, by Course

Course	Publisher	Title
Non-college prep Mathematics	Houghton Mifflin Harcourt Houghton Mifflin Harcourt Pearson	*Algebra 1* *Geometry* *Algebra 1*
Formal/College-prep Mathematics Level 1	Houghton Mifflin Harcourt Pearson McGraw-Hill	*Algebra 1* *Algebra 1* *Algebra 1*
Formal/College-prep Mathematics Level 2	Houghton Mifflin Harcourt Pearson	*Geometry* *Geometry*
Formal/College-prep Mathematics Level 3	Houghton Mifflin Harcourt Pearson	*Algebra 2* *Algebra 2*
Formal/College-prep Mathematics Level 4	Cengage Learning McGraw-Hill	*Precalculus with Limits: A Graphing Approach* *Advanced Mathematical Concepts: Precalculus with Applications*
Courses that might qualify for college credit	Pearson Cengage Learning	*Calculus: Graphical, Numerical, Algebraic* *Calculus of a Single Variable*

Table 33 shows the publication year of commercially published instructional materials. In 2012, over half of high school mathematics classes were using materials published prior to 2007. Only 15 percent of classes were using materials published in the past two years.

Table 33
Publication Year of Instructional Materials Used in High School Mathematics Classes

	Percent of Classes[†]							
	All		Informal Review		Formal Required		Formal Advanced	
2006 or earlier	53	(1.9)	54	(5.4)	48	(2.6)	57	(2.4)
2007–09	32	(1.6)	37	(5.1)	36	(2.3)	28	(2.0)
2010–12	15	(1.1)	9	(2.7)	16	(1.6)	15	(1.4)

[†] Only classes using commercially published textbooks/programs were included in these analyses.

It is interesting to note that while national experts in mathematics education are often critical of textbook quality,[8] most high school mathematics teachers consider their instructional materials to be of relatively high quality, as those in about 80 percent of classes rated their materials as good or better (see Table 34).

Table 34
Perceived Quality of Instructional Materials Used in High School Mathematics Classes

	Percent of Classes[†]							
	All		Informal Review		Formal Required		Formal Advanced	
Very Poor	1	(0.4)	0	---[‡]	1	(0.8)	1	(0.6)
Poor	5	(0.8)	4	(2.2)	6	(1.5)	4	(1.2)
Fair	16	(1.3)	25	(6.6)	18	(2.4)	12	(2.0)
Good	34	(2.7)	41	(7.9)	36	(3.8)	30	(3.5)
Very Good	36	(2.4)	29	(6.0)	33	(3.2)	41	(4.1)
Excellent	8	(1.1)	1	(1.0)	6	(1.7)	12	(1.8)

[†] Only classes using commercially published textbooks/programs were included in these analyses.
[‡] No teachers in the sample selected this response option. Thus, it is not possible to calculate the standard error of this estimate.

Despite these strong ratings of quality, only 67 percent of high school mathematics classes address three-fourths or more of their instructional materials. This finding may result from publishers' efforts to meet as many state and district criteria as possible by including all of the content anyone might seek (see Table 35). Interestingly, about 1 in 5 high school mathematics classes spend less than 25 percent of their instructional time using the commercially published textbook or program materials (see Table 36).

[8] For example, American Association for the Advancement of Science (2000). *Middle grades mathematics textbooks: A benchmarks-based evaluation.* Washington, DC: Author.

Table 35
Percentage of Instructional Materials
Covered during High School Mathematics Courses

	Percent of Classes†			
	All	Informal Review	Formal Required	Formal Advanced
Less than 25 percent	1 (0.3)	0 (0.5)	1 (0.5)	1 (0.4)
25–49 percent	6 (1.3)	11 (5.4)	3 (1.2)	8 (2.0)
50–74 percent	26 (2.2)	41 (8.8)	14 (2.8)	31 (3.3)
75 percent or more	67 (2.3)	48 (9.4)	83 (2.9)	60 (3.6)

† Only classes using commercially published textbooks/programs were included in these analyses

Table 36
Percentage of Instructional Time Spent
Using Instructional Materials during High School Mathematics Courses

	Percent of Classes†			
	All	Informal Review	Formal Required	Formal Advanced
Less than 25 percent	21 (2.2)	21 (7.3)	16 (2.8)	25 (3.4)
25–49 percent	13 (1.7)	18 (6.7)	12 (2.1)	14 (2.4)
50–74 percent	20 (1.8)	30 (7.9)	23 (3.6)	17 (2.2)
75 percent or more	45 (2.7)	31 (8.9)	49 (4.2)	45 (3.8)

† Only classes using commercially published textbooks/programs were included in these analyses

A similar story emerges from responses to questions asking teachers to describe how they used their textbook/program in their most recent unit. As can be seen in Table 37, teachers in 56 percent of high school mathematics classes indicate that they supplemented their textbook/ program, a practice that is more common in informal review courses than formal advanced courses. Teachers in 51 percent of high school mathematics classes indicate that they picked what was important from the materials and skipped the rest. Still, in the majority of high school mathematics classes, teachers use the textbook/program to guide the overall structure and content emphasis of their units.

Table 37
Ways High School Mathematics Teachers
Substantially[†] Used their Instructional Materials in the Most Recent Unit

	Percent of Classes[‡]			
	All	Informal Review	Formal Required	Formal Advanced
You used the textbook/program to guide the overall structure and content emphasis of the unit	74 (1.6)	72 (4.6)	79 (2.5)	71 (2.4)
You followed the textbook/program to guide the detailed structure and content emphasis of the unit	57 (1.6)	55 (4.6)	59 (2.4)	55 (2.3)
You incorporated activities (e.g., problems, investigations, readings) from other sources to supplement what the textbook/program was lacking	56 (2.0)	66 (5.0)	58 (3.0)	52 (2.6)
You picked what is important from the textbook/program and skipped the rest	51 (1.6)	48 (4.9)	48 (2.9)	55 (2.1)

[†] Includes those responding 4 or 5 on a 5-point scale ranging from 1 "not at all" to 5 "to a great extent."
[‡] Only classes using commercially published textbooks/programs in the most recent unit were included in these analyses.

Asked why they supplement their textbook/program, teachers in nearly all high school mathematics classes indicated they do so to provide students with additional practice and to help students at different levels of achievement learn targeted ideas (see Table 38). Similarly, in about half of mathematics classes, teachers supplement to prepare students for standardized tests, although this practice is much more common for informal review classes than formal advanced classes. Teachers in over 40 percent of informal review and formal required mathematics classes use supplemental activities as indicated in their pacing guide; teachers in only about a quarter of formal advanced classes supplement for this reason.

Table 38
Reasons Why High School
Mathematics Instructional Materials Are Supplemented

	Percent of Classes[†]			
	All	Informal Review	Formal Required	Formal Advanced
Supplemental activities were needed to provide students with additional practice	94 (1.3)	91 (5.1)	96 (1.6)	92 (2.5)
Supplemental activities were needed so students at different levels of achievement could increase their understanding of the ideas targeted in each activity	91 (1.6)	93 (4.2)	91 (2.4)	90 (2.2)
Supplemental activities were needed to prepare students for standardized tests	55 (2.6)	70 (6.2)	61 (3.7)	45 (3.6)
Your pacing guide indicated that you should use supplemental activities	36 (2.2)	43 (6.8)	45 (3.5)	26 (2.9)

[†] Only classes using commercially published textbooks/programs in the most recent unit and whose teachers reported supplementing some activities were included in these analyses.

Teachers were also asked why they skipped parts of their textbook/program. As can be seen in Table 39, teachers in 79 percent of high school mathematics classes skip activities because they have other ones that work better. In two-thirds of mathematics classes, teachers skip activities

because the ideas are not in their pacing guides/state standards. Skipping activities because the students already knew the content or the activities were too difficult are also fairly common.

Table 39
Reasons Why Parts of High School Mathematics Instructional Materials Are Skipped

	Percent of Classes[†]							
	All		Informal Review		Formal Required		Formal Advanced	
You have different activities for those mathematical ideas that work better than the ones you skipped	79	(2.2)	88	(4.6)	81	(3.2)	76	(3.4)
The mathematical ideas addressed in the activities you skipped are not included in your pacing guide and/or current state standards	66	(3.0)	74	(6.2)	72	(4.2)	58	(3.6)
The activities you skipped were too difficult for your students	56	(2.6)	69	(6.3)	65	(3.9)	43	(3.5)
Your students already knew the mathematical ideas or were able to learn them without the activities you skipped	54	(2.9)	55	(8.0)	55	(4.3)	54	(3.9)
You did not have the materials needed to implement the activities you skipped	31	(2.8)	34	(7.5)	39	(4.3)	22	(3.0)

[†] Only classes using commercially published textbooks/programs in the most recent unit and whose teachers reported skipping some activities were included in these analyses.

Facilities and Equipment

Teachers were presented with a list of instructional technologies and asked about their availability in the randomly selected class. The three response options were:

- Do not have one per group available;
- At least one per group available upon request or in another room; and
- At least one per group located in your classroom.

As can be seen in Table 40, formal advanced mathematics classes are much more likely than informal review and formal required mathematics classes to have access to graphing calculators and probes for data collection. Access to other instructional technologies, such as personal computers and the Internet, is similar across the three types of classes.

Table 40
Availability† of Instructional Technologies in High School Mathematics Classes

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
Graphing calculators	84	(1.7)	73	(5.8)	78	(3.1)	92	(1.5)
Scientific calculators	74	(1.7)	77	(5.9)	70	(3.2)	76	(2.4)
Internet access	71	(2.0)	73	(5.4)	71	(2.9)	71	(3.3)
Four-function calculators	61	(2.0)	63	(6.3)	60	(3.3)	62	(2.5)
Personal computers, including laptops	59	(2.4)	59	(5.5)	58	(3.1)	59	(3.8)
Classroom response system or "Clickers" (handheld devices used to respond electronically to questions in class)	45	(2.7)	38	(7.1)	46	(3.3)	45	(3.9)
Probes for collecting data (e.g., motion sensors, temperature probes)	27	(2.3)	17	(4.7)	21	(2.8)	34	(3.6)
Hand-held computers (e.g., PDAs, tablets, smartphones, iPads)	17	(1.4)	18	(4.4)	17	(2.5)	17	(1.9)

† Includes only those rating the availability as at least one per group available, either in the classroom, upon request, or in another room.

Although the majority of high school mathematics classes have access to graphing calculators, 31 percent expect students to provide their own (see Table 41). This expectation is much more common in formal advanced classes than other mathematics classes.

Table 41
Expectations that Students will Provide their Own Instructional Technologies

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
Scientific calculators	37	(2.0)	32	(5.3)	41	(3.6)	36	(2.6)
Graphing calculators	31	(2.0)	10	(3.3)	16	(2.3)	50	(3.0)
Four-function calculators	23	(1.9)	25	(5.3)	26	(3.1)	21	(2.7)
Laptop computers	7	(1.1)	5	(2.3)	5	(1.3)	10	(1.7)
Hand-held computers	6	(0.9)	7	(3.1)	6	(1.2)	7	(1.4)

When asked about the adequacy of resources for instruction, teachers in the majority of high school mathematics classes rated their measurement tools, instructional technology, and access to consumable supplies as adequate (see Table 42). On a composite variable created from these items titled "Adequacy of Resources for Instruction," mean scores were similar across course types (see Table 44).

Table 42
High School Mathematics Classes with Adequate† Resources for Instruction

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
Measurement tools (e.g., protractors, rulers)	70	(1.5)	77	(4.2)	68	(2.3)	71	(2.1)
Instructional technology (e.g., calculators, computers, probes/sensors)	68	(1.7)	69	(4.5)	65	(2.2)	72	(2.5)
Consumable supplies (e.g., graphing paper, batteries)	66	(1.8)	64	(5.0)	62	(2.8)	69	(2.2)
Manipulatives (e.g., pattern blocks, algebra tiles)	43	(1.7)	50	(4.8)	38	(2.6)	46	(2.7)

† Includes those responding 4 or 5 on a 5-point scale ranging from 1 "not adequate" to 5 "adequate."

Table 43
Class Mean Scores on the
Adequacy of Resources for Instruction Composite

	Mean Score	
All	70	(0.8)
Informal Review	72	(2.3)
Formal Required	68	(1.1)
Formal Advanced	72	(1.1)

FACTORS AFFECTING HIGH SCHOOL MATHEMATICS INSTRUCTION

Teachers were asked about factors that affect instruction in their randomly selected class. As can be seen in Table 44, teachers in the majority of high school mathematics classes think that most of the factors promote effective instruction. Factors seen as promoting effective instruction in many mathematics classes include principal support, college entrance requirements, pacing guides, and curriculum frameworks. Student motivation and abilities, parent expectations and involvement, and students' reading abilities are more likely to be seen as promoting effective instruction in advanced mathematics classes than other mathematics classes, perhaps because advanced mathematics is often an elective taken only by the higher-achieving students in schools. Community views on mathematics instruction and state/district testing/accountability policies are seen as promoting effective instruction in a minority of high school mathematics classes.

Table 44
Factors Promoting† Effective Instruction in High School Mathematics Classes

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
Principal support	74	(1.9)	68	(6.0)	75	(2.7)	75	(2.7)
College entrance requirements	66	(2.1)	52	(7.0)	62	(3.3)	75	(2.9)
District/Diocese/School pacing guides	63	(2.2)	64	(6.4)	62	(3.0)	63	(3.2)
District/Diocese curriculum frameworks‡	63	(2.2)	59	(5.7)	65	(3.1)	61	(3.3)
Time for you to plan, individually and with colleagues	61	(2.3)	53	(6.5)	64	(3.1)	60	(3.7)
Current state standards	58	(1.9)	58	(5.6)	64	(2.7)	52	(3.2)
Time available for your professional development	55	(2.0)	44	(6.1)	57	(3.3)	56	(3.4)
Students' motivation, interest, and effort in mathematics	54	(2.5)	39	(5.4)	49	(3.4)	64	(4.1)
Teacher evaluation policies	54	(2.0)	55	(5.6)	53	(2.8)	56	(3.3)
Textbook/program selection policies	52	(2.1)	47	(6.0)	48	(2.8)	59	(3.6)
Parent expectations and involvement	46	(2.1)	34	(5.5)	40	(3.3)	57	(3.8)
District/Diocese testing/accountability policies‡	45	(2.2)	57	(6.4)	44	(2.9)	42	(3.6)
Students' reading abilities	44	(2.5)	27	(4.8)	38	(3.8)	55	(3.8)
Community views on mathematics instruction	39	(2.3)	32	(6.0)	37	(3.4)	44	(3.5)
State testing/accountability policies‡	38	(1.9)	46	(5.8)	39	(2.9)	35	(3.5)

† Includes those responding 4 or 5 on a 5-point scale ranging from 1 "inhibits effective instruction" to 5 "promotes effective instruction."

‡ Item presented only to public and catholic school teachers.

The teacher survey also included a series of items about technology-related issues. Teachers were asked to indicate how great a problem each posed for instruction in their randomly selected class. As can be seen in Table 45, these resources are generally not seen as problematic in high school mathematics classes.

Table 45
Extent to which Technology Quality Is a Serious Problem for Instruction in High School Mathematics Classes

	Percent of Classes							
	All		Informal Review		Formal Required		Formal Advanced	
Lack of availability of appropriate computer software	12	(1.4)	11	(4.0)	13	(2.0)	11	(2.2)
Old age of computers	9	(1.4)	11	(3.7)	11	(2.0)	7	(1.9)
Lack of availability of technology support	8	(1.2)	4	(1.9)	9	(2.0)	9	(1.7)
Lack of access to computers	8	(1.3)	7	(3.1)	10	(1.9)	6	(2.0)
Slow speed of the Internet connection	6	(1.2)	7	(3.7)	9	(2.0)	4	(1.2)
Unreliability of the Internet connection	5	(1.1)	3	(2.0)	7	(1.7)	3	(1.1)
Lack of access to the Internet	3	(0.8)	3	(1.6)	5	(1.4)	2	(0.9)

Composites from these two series of questionnaire items were created to summarize the extent to which various factors support effective instruction. The means are shown in Table 46. Overall, these data indicate that the climate is generally supportive for high school mathematics

instruction. The positive influence of stakeholder support on effective instruction is seen as stronger in advanced mathematics classes than in formal required or informal review classes.

Table 46
Class Mean Scores for the Factors Affecting Instruction Composites

	Mean Score			
	All	Informal Review	Formal Required	Formal Advanced
Extent to which School Support Promotes Effective Instruction	67 (1.2)	63 (3.2)	67 (2.0)	67 (1.8)
Extent to which the Policy Environment Promotes Effective Instruction	65 (0.8)	68 (2.3)	65 (1.2)	64 (1.6)
Extent to which Stakeholders Promote Effective Instruction	59 (1.3)	49 (2.5)	55 (1.8)	65 (2.1)
Extent to which IT Quality is Problematic for Instruction	19 (1.0)	16 (2.7)	22 (1.5)	16 (1.3)

SUMMARY

Nearly all high school mathematics classes are taught by teachers who are white, and over half are taught by teachers with more than 10 years of teaching experience. Three-quarters of classes are led by a teacher with a degree in mathematics or mathematics education. However, almost half of all high school mathematics classes are taught by teachers who have not taken a college course in mathematics within the past 10 years. Teachers of high school mathematics classes tend to feel well prepared to teach algebraic thinking, the number system and operations, and functions, and less well prepared to teach statistics and probability and discrete mathematics. In addition, although mathematics teachers hold a number of beliefs about teaching and learning that are in alignment with what is known about effective mathematics instruction (e.g., it is good for students to share their thinking and reasoning), they also hold views that are more consistent with traditional instruction. For example, teachers in about 80 percent of high school mathematics classes believe that students should be provided with definitions for new vocabulary at the beginning of instruction on an idea.

When asked about their professional development experiences, teachers in the vast majority of high school mathematics classes have participated in mathematics-focused professional development in the last three years. However, only one-third of classes are taught by a teacher with extensive mathematics-focused professional development (more than 35 hours) in that time period. Compared to teachers of more advanced courses, teachers of informal review classes have had more opportunities to try out what they learned in the classroom and then talk about it as a part of the professional development.

Data on mathematics courses indicate that nearly all students in the nation have access to one or more formal/college-prep mathematics courses. An overwhelming majority of high schools offer higher level formal/college courses as well. Female students are just as likely as male students to take mathematics courses, with somewhat fewer females than males in informal review classes.

Students from race/ethnic groups historically underrepresented in mathematics are less likely to take more advanced mathematics classes.

Data on instruction indicate that high school mathematics instruction relies heavily on lecture and discussion, with students often completing textbook/worksheet problems. However, the data also indicate that students are engaged in justifying their method for solving a problem fairly regularly. In addition, instruction in informal review classes appears to be much more driven by high-stakes testing than formal advanced courses.

The use of calculators is common in high school mathematics classes. Over 60 percent of high school mathematics classes use a single commercially published textbook or program, although over 30 percent cover less than three-quarters of the material in their textbook.

APPENDIX

Table A-1
Definitions of Grade 9–12 Mathematics Course Types

Course Type	Classification	Example Courses
Non-College Prep	Informal Review	Developmental Math; High School Arithmetic; Remedial Math; General Math; Vocational Math; Consumer Math; Basic Math; Business Math; Career Math; Practical Math; Essential Math; Pre-Algebra; Introductory Algebra; Algebra 1 Part 1; Algebra 1A; Math A; Basic Geometry; Informal Geometry; Practical Geometry
Formal/College-Prep Level 1	Formal Required	Algebra 1; Integrated Math 1; Unified Math I; Algebra 1 Part 2; Algebra 1B; Math B
Formal/College-Prep Level 2	Formal Required	Geometry; Plane Geometry; Solid Geometry; Integrated Math 2; Unified Math II; Math C
Formal/College-Prep Level 3	Formal Advanced	Algebra 2; Intermediate Algebra; Algebra and Trigonometry; Advanced Algebra; Integrated Math 3; Unified Math III
Formal/College-Prep Level 4	Formal Advanced	Algebra 3; Trigonometry; Pre-Calculus; Analytic/Advanced Geometry; Elementary Functions; Integrated Math 4; Unified Math IV; Calculus (not including college level/AP); any other College Prep Senior Math with Algebra 2 as a prerequisite
Courses that might qualify for college credit	Formal Advanced	Advanced Placement Calculus (AB, BC); Advanced Placement Statistics; IB Mathematics standard level; IB Mathematics higher level; concurrent college and high school credit/dual enrollment

Table A-2
Class Mean Scores for Composites

	Mean Score		
	Informal Review	Formal Required	Formal Advanced
Perceptions of Preparedness to Teach Mathematics Content	79 (1.5)	83 (0.6)	86 (0.6)
Perceptions of Preparedness to Encourage Students' Interest in Math	75 (2.4)	72 (1.6)	77 (1.3)
Perceptions of Preparedness to Teach Students from Diverse Backgrounds	58 (3.1)	56 (1.4)	57 (1.2)
Perceptions of Preparedness to Implement Instruction in Particular Unit	83 (1.6)	85 (0.7)	86 (0.7)
Quality of Professional Development	66 (2.9)	66 (1.7)	61 (1.2)
Extent to which PD/Coursework Focused on Student-Centered Instruction	53 (2.4)	50 (1.2)	47 (1.3)
Curriculum Control	56 (4.3)	44 (2.1)	57 (2.0)
Pedagogical Control	86 (2.2)	85 (1.1)	91 (0.8)
Reform-Oriented Instructional Objectives	74 (1.4)	78 (0.6)	80 (0.6)
Use of Reform-Oriented Teaching Practices	66 (1.6)	69 (0.9)	67 (0.8)
Use of Instructional Technology	30 (3.6)	19 (1.5)	21 (1.1)
Adequacy of Resources for Instruction	72 (2.3)	68 (1.1)	72 (1.1)
Extent to which Stakeholders Promote Effective Instruction	49 (2.5)	55 (1.8)	65 (2.1)
Extent to which the Policy Environment Promotes Effective Instruction	68 (2.3)	65 (1.2)	64 (1.6)
Extent to which School Support Promotes Effective Instruction	63 (3.2)	67 (2.0)	67 (1.8)
Extent to which IT Quality is Problematic for Instruction	16 (2.7)	22 (1.5)	16 (1.3)